Mining a medieval social network by kernel SOM and related methods

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Joint work with **Fabrice Rossi**, INRIA, Rocquencourt, France
and **Quoc-Dinh Truong**, IRIT, Toulouse, France

Journées MASHS, June 5*th*, 2008

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A multidisciplinary project: "Graph-Comp"

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Institut de Recherche en Informatique
de Toulouse
(Univ. de Toulouse & CNRS)

Laboratoire d'Informatique
de Nantes Atlantique
(Univ. de Nantes & CNRS)

INRIA Rocquencourt
Projet AxIS

A multidisciplinary project: "Graph-Comp"

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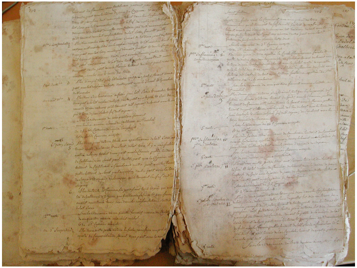

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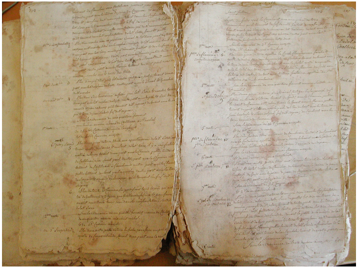

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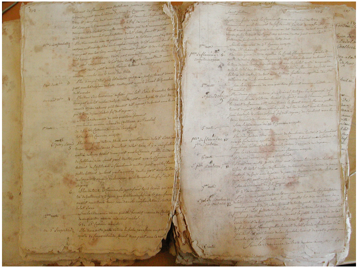

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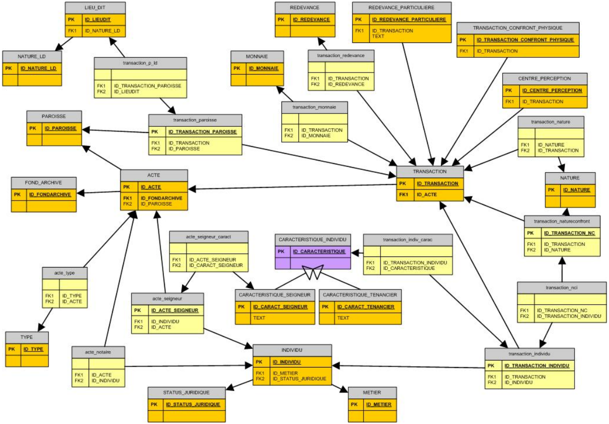
This corpus interests the historians because:

- only a few documents from middle ages deal with peasants' life;
- it permits to study *a priori* the evolution of the social network before and after the Hundred Years' War.

Methodology

Each contract of the corpus is recorded in a database (still to be finished) thought by Fabien Picarougne:



Parts of this database can be accessed on the web site http://graphcomp.univ-tlse2.fr/.

A large graph for the medieval social network

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- the graph thus have weights $(w_{i,j})_{i,j=1\dots,n}$ which are the number of contracts satisfying one of these conditions. They are such that:
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Providing tools to help historians understanding the structure of this social network.

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But ! How to visualize and/or simplify this graph to interpret it ?

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Graph drawing aims at the arrangement of the vertices and edges in order to make the representation of the graph understandable and aesthetics. See Graph Visualization Software References website:

http://www.polytech.univ-nantes.fr/GVSR/

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Here, Tulip.
Enables force-directed algorithms:

gradient-descent minimization of an energy function.

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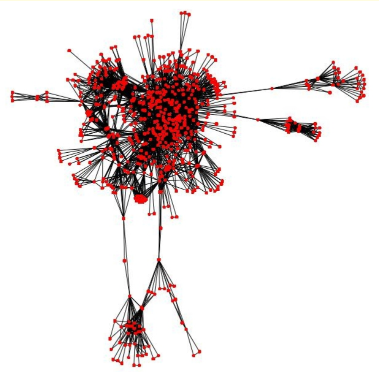
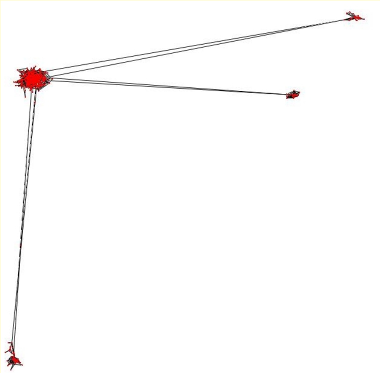
Representation of the medieval network by force-directed algorithms



"GEM"



"Spring Electrical"

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Aims of the clustering

We want to underline homogeneous social groups that are fewly connected to each others

[Newman and Girvan, 2004]: *"reducing [the] level of complexity [of a network] to one that can be interpreted readily by the human eye, will be invaluable in helping us to understand the large-scale structure of these new network data"*

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Review on clustering of the vertices of a graph in [Schaeffer, 2007]:

- How to measure the quality of a graph clustering?
- Presentation of global or local algorithms based on
  - a similarity measure and the adaptation of a clustering algorithm to similarity data;
  - mapping of the graph on a euclidean space;
  - the minimization of a cluster fitness measures.

  Several kinds:
  - batch
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Spectral clustering [von Luxburg, 2007]

For a graph with vertices $V = \{x_1, \ldots, x_n\}$ having positive weights $(w_{i,j})_{i,j=1,\ldots,n}$ **Laplacian**: $L = (L_{i,j})_{i,j=1,\ldots,n}$ where

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Graph cut optimization

If the graph is connected, clustering the vertices into k groups $A_1, \ldots, A_k$ that minimize

$$Cut(A_1, \ldots, A_k) = \sum_{i=1}^{k} \sum_{j \in A_i,\ j' \notin A_i} w_{i,i'}$$

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$\Leftrightarrow$ find $(h_1, \ldots, h_k) \in \prod_{i=1}^{k} \left\{0, \frac{1}{\sqrt{|A_i|}}\right\}^n$ that minimizes

$$\sum_{i=1}^{k} h_i^T L h_i \text{ subject to } (h_1 \ldots h_k)(h_1 \ldots h_k)^T = \mathbb{I}_n$$

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$\simeq$ find $h_1, \ldots, h_k \in \mathbb{R}^n$ that minimize (continuous approximation)

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Spectral clustering on medieval social network

Algorithm

1. Compute the eigenvectors, $v_1, \ldots, v_k \in \mathbb{R}^n$ of L associated with the k smallest positive eigenvalues.

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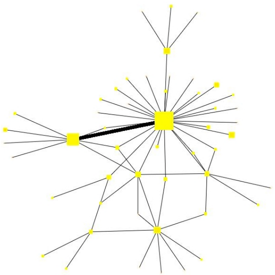
Representation of the clustering (k-means, 50 clusters + force directed algorithm):



2 big clusters of central people highly connected;

Identification of individuals that help to connect the network;

isolated individuals around.

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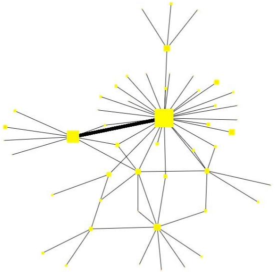

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But: Size of the biggest cluster: 268 !!

16 clusters have size 1

more than 50% of the clusters have a size less than 2

A regularized version of the Laplacian: the heat kernel

From the diffusion matrix to the heat kernel

Diffusion matrix: for $\beta > 0$, $K^\beta = e^{-\beta L}$.

$\Rightarrow$

$$\begin{array}{rccc} k^\beta : & V \times V & \rightarrow & \mathbb{R} \\ & (x_i, x_j) & \rightarrow & K^\beta_{i,j} \end{array}$$

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Intuitive interpretation: $k^\beta(x_i, x_j) \simeq$ quantity of energy accumulated in x_j after a given time if energy is injected in x_i at time 0 and if diffusion is done along the edges (β control the intensity of the diffusion).

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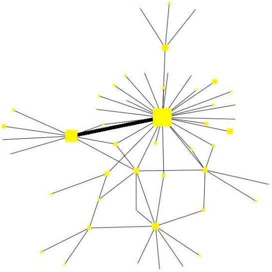
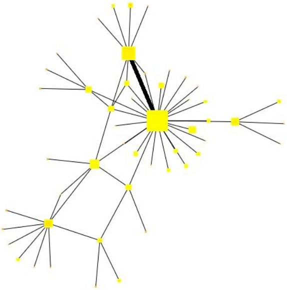
Mapping of the graph on a euclidean space: k^β is the scalar product associated with the mapping

$$\phi : x_i \in V \rightarrow (v_1 \ldots v_n)_i \in \mathbb{R}^n_\lambda$$

where $(v_l)_l$ are the eigenvectors of L and $\mathbb{R}^n_\lambda$ denotes the n-dimensional space with norm weighted by $(e^{-\beta\lambda_l})_l$ ($\lambda \equiv$ eigenvalues of L).





Spectral Clustering	Kernel *k*-means
Max size: 268	242
Nb of clusters of size 1: 16	17
Median size: 2	2

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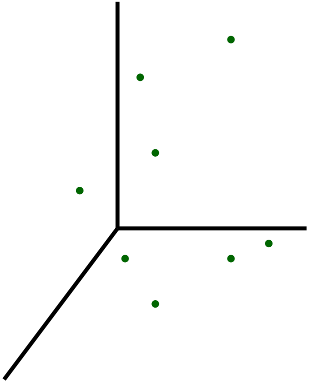
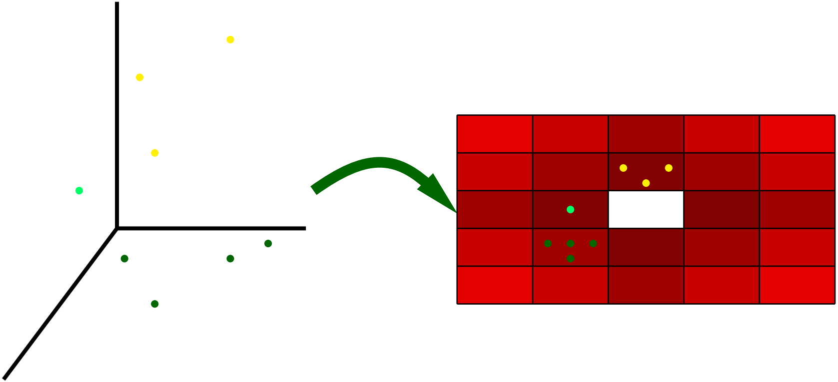
General principle of SOM for graphs



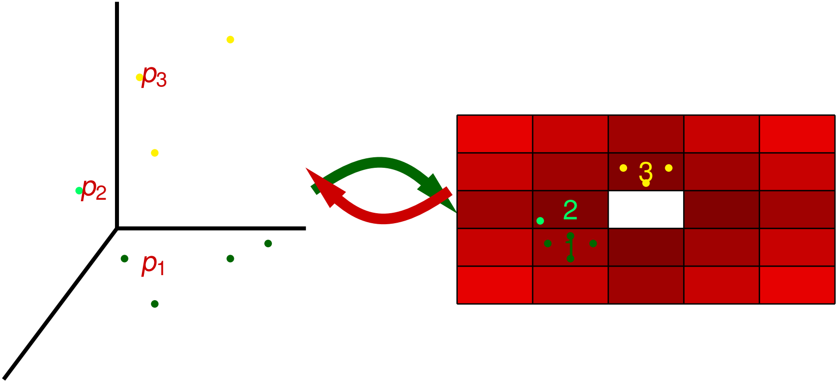
The vertices of the graph are mapped on a euclidean space (by L: "spectral SOM" or by K: "kernel SOM").

General principle of SOM for graphs



Each vertex x_i is affected to a neuron (a cluster) of the Kohonen map, $f(x_i)$. Neurons are related to each others by a neighborhood relationship ("distance": d).

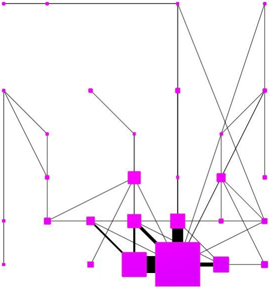
General principle of SOM for graphs

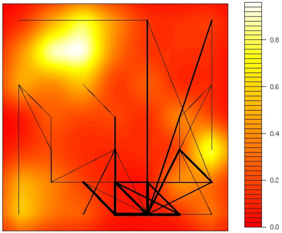


Each neuron j of the map is represented by a prototype p_j.
Couples (j, p_j) and $(x_i, f(x_i))$ depend from each others and are iteratively updated in order to approach the minimization of the energy of the map:

$$\mathcal{E}^n = \sum_{j=1}^{n} \sum_{i=1}^{M} h(d(f(x_j), i)) \|\phi(x_i) - p_j\|^2.$$

Spectral SOM



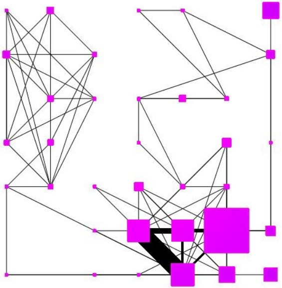


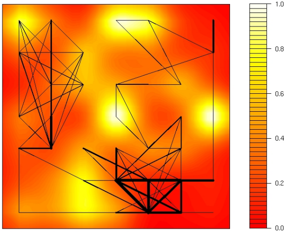
Number of clusters 29
Number of clusters of size 1 11
Maximum size of the clusters 325
Median size 2

Q-modularity: $\sum_{i=1}^{k}(e_i - a_i^2)$

$Q_{\text{modul}} = 0.433$
(vs 0.420 & 0.425 for clusterings)

Kernel SOM [Boulet et al., 2008]



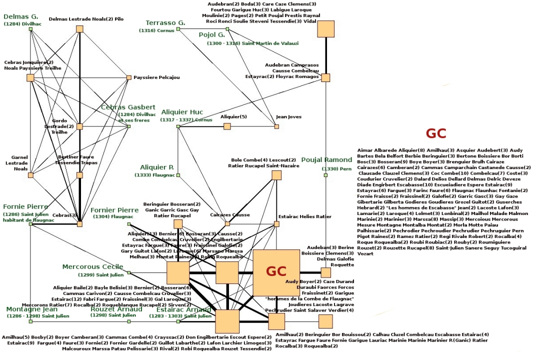


Number of clusters 35
Number of clusters of size 1 13
Maximum size of the clusters 255
Median size 3

Q-modularity: $\sum_{i=1}^{k}(e_i - a_i^2)$

$$Q_{\text{modul}} = 0.551$$

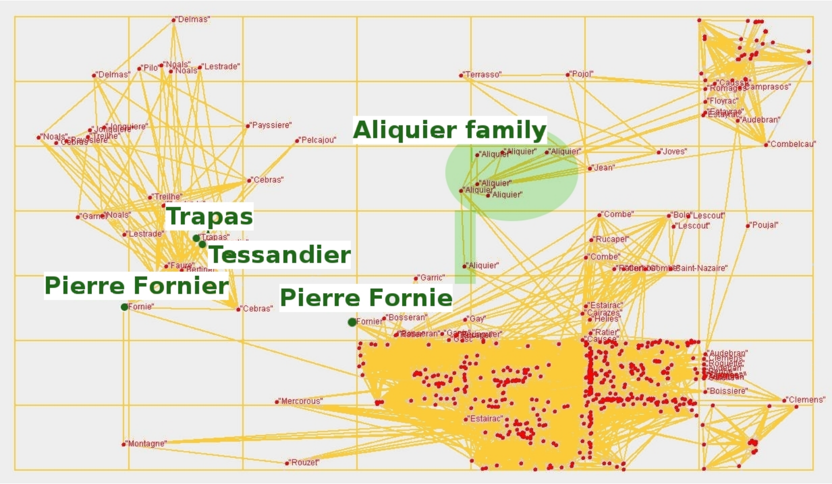
1. Dates
2. Lieux

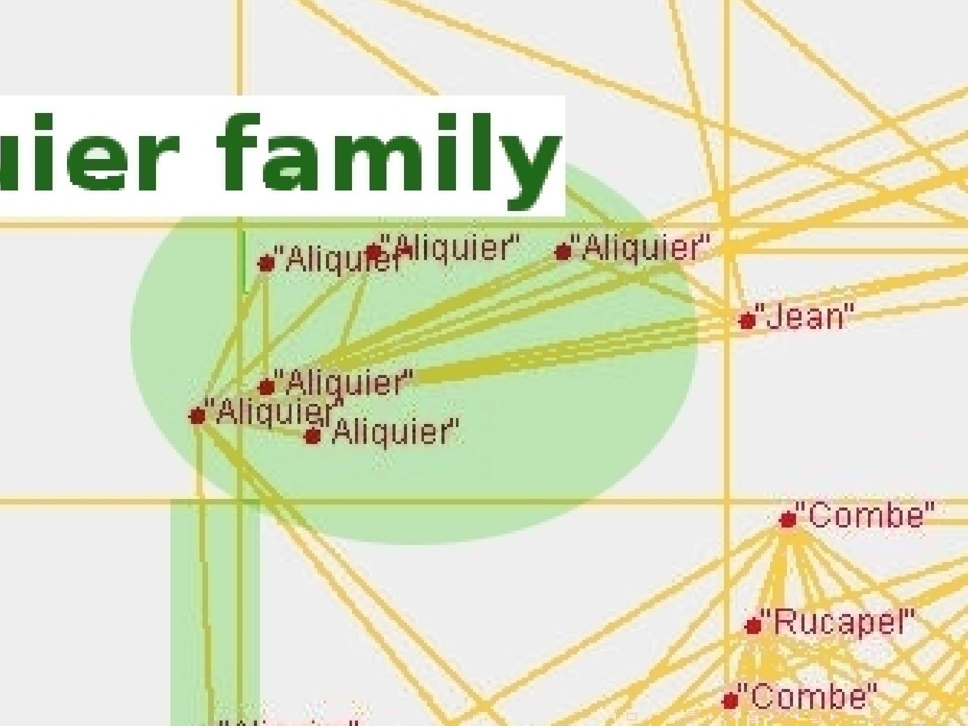


Force directed algorithm for clustered graphs [Truong et al., 2007, Truong et al., 2008]

La Suite...

By adding constrains on force-directed algorithms




uier family
"Aliquier"
"Aliquier"
"Aliquier"
"Jean"
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"Combe"
"Rucapel"
"Combe"


"Ratier"
"Causse"
"Estairac"

Perspectives

Several perspectives:

- Improving the global representation of the network (hierarchical algorithms, improving algorithms for clustered graphs drawing, others algorithms such as simulated annealing, . . .)

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- Improving the global representation of the network (hierarchical algorithms, improving algorithms for clustered graphs drawing, others algorithms such as simulated annealing, . . .)
- Understanding the evolution of the social network through time (before/during/after Hundred Years' War): specific tools have to be built in order to
  - understand what become the dominant families ("Aliquier", "Fornie", . . .),
  - make a comparison despite the fact that the vertices are not the same.

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